View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Elsevier - Publisher Connector

ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Agriculture and Agricultural Science Procedia 7 (2015) 35 – 39
Agriculture and Agricultural Science Procedia

Farm Machinery and Processes Management in Sustainable Agriculture, 7th International Scientific Symposium

Mathematical modelling of the process of renewal of the fleet of combine harvesters

Volodymyr Bulgakov[a], Valerii Adamchuk[b], Margus Arak[c], Jüri Olt[c,]*

[a]*National University of Life and Environmental Sciences of Ukraine, 15 Heroyiv Oborony Str., Kyiv, 03041, Ukraine*
[b]*Institute for Agricultural Engineering and Electrification, National Scientific Centre, 11 Vokzalna Str., Glevakha-1, Kiev Region, 08631, Ukraine*
[c]*Institute of Technology, Estonian University of Life Sciences, 56 Kreutzwaldi Str., Tartu, 51014, Estonia*

Abstract

The new optimization task within the integrated model of the renewal of the fleet of harvesters has been considered in this paper. The aim of the study is generation of the mathematical model for the renewal of the combine harvester fleet on the basis of integral equations with an unknown lower limit of integration. The integro-functional equations for main trajectories have been obtained, and the qualitative behaviour of main trajectories depending on the rates of scientific and technological advancement has been studied.

© 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the Centre wallon de Recherches agronomiques (CRA-W)

Keywords: agricultural machinery; combine harvester; fleet; model; optimization

1. Introduction

In order to solve the problem at hand, we have used the integrated model theory of the systems with controlled memory, which have a number of advantages in comparison to other models that are widely used at present. Integral dynamic models are flexible enough to be applied, and after being modified accordingly, they can be used for a wide range of purposes and practical tasks. Depending on the tasks that have been set, a number of model ratios, which

* Corresponding author. Tel.: +372-7-313-358; fax: +372-7-313-334.
E-mail address: jyri.olt@emu.ee

2210-7843 © 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Peer-review under responsibility of the Centre wallon de Recherches agronomiques (CRA-W)
doi:10.1016/j.aaspro.2015.12.027

are insignificant in this particular case, might not be used or, on the contrary, the new correlations, which describe any given peculiarities of the course of technological processes, can be included into the equation system. If we take into account the increasing deficit in labour and fuel resources, the need to use models like the one described above for managing the processes of the replacement and renewal of technical equipment becomes obvious. A good overview can be found in Bochtis et al., (2007 and 2014), Edwards and Boehlje (1980), Gao et al., (1985), Gunnarson et al., (2009), Miu et al., (1997) and Trollope (1982).

2. **Materials and Methods**

The main task is to define the composition of a fleet of combine harvesters in order to ensure the performance of all tasks arising from the structure of cultivated areas, aimed at harvesting grain crops within the specified agrotechnical time periods with minimum material costs and existing labour resources.

At first, we are going to set the input parameters that are essential for modelling: Thereby, I is the initial set of the types of grain crops and grain legume crops that have to be harvested with combine harvester; i is the crop type, $i \in I$, $|I| = n$; J is the set of combine harvester makes; j is the number of a combine harvester makes, $j \in J$, $|J| = m$; τ is the year of purchase (manufacture) of a combine harvester, $\tau \in [\tau_0, t_0]$; $[t_0, T]$ is the scheduled calculation interval in years; $[\tau_0, t_0]$ is the interval of development of the fleet of combine harvesters; $\beta_{ij}(\tau,t)$ is the output capacity of harvesters of j make, manufactured or purchased in τ, which harvest i crop in t year; $P_{ij}(\tau)$ is the amount of staff that provides maintenance to the j make, which was manufactured or purchased in τ, which harvests i crop; $x_{ij}(\tau)$ is the number of combine harvesters of the j make, manufactured or purchased in τ, which harvest i grain crop, $\tau \in [\tau_0,t_0]$; $a_j(t)$ is time limit for writing off the combine harvesters of the j make: if a harvester was manufactured or purchased in τ, the combine harvester is amortized at $\tau < a_j(t)$, at $\tau \geq a_j(t)$ it is used in an operating cycle at $t \in [t_0, T]$; $S_i(t)$ is the rated area of i grain crop or grain legume crop in t, $t \in [t_0, T]$; $P(t)$ is the availability of machine operators in year t.

The demand for combine harvester should be determined proceeding from the need to harvest all kinds of grain crops and grain legume crops per year t, $t \in [t_0, T]$:

$$S_i(t) = \sum_{j=1}^{m_i} \int_{a_{j(t)}}^{t} \beta_{ij}(\tau,t) x_{ij}(\tau) d\tau, \tag{1}$$

$$i = \overline{1,n}$$

where m_i is the number of the makes of harvester-threshers used for harvesting i crops.

The demand for labour force required for performing the specific types of work and operations is determined on the basis of the following equation:

$$P(t) = \sum_{i=1}^{n} \sum_{j=1}^{m_i} \int_{a_{j(t)}}^{t} P_{ij}(\tau) x_{ij}(\tau) d\tau \,. \tag{2}$$

The system of integral equations (1) – (2) is an integral dynamic model with controlled memory. It describes the processes of replacing and renewing the fleet of combine harvesters taking into account scientific and technological progress, since the output capacity of a combine harvester $\beta(\tau,t)$ depends on the year of its manufacture τ, while this function $\beta(\tau,t)$ is non-decreasing, but in the majority of cases it decreases by τ. The functions $S_i(t)$, $P(t)$, $\beta_{ij}(\tau,t)$, $P_{ij}(\tau)$ will always be regarded as pre-set, $t \in [t_0,T]$, $\tau \in [\tau_0,t_0]$.

The functions $x_{ij}(t)$, $x_{ij} \geq = 0$, $i = \overline{1,n}$, $j = \overline{1,m_i}$ are always sought-for, while the functions $a_j(t) < t$, $j = \overline{1,m_i}$ can be pre-set and sought-for, depending on the problem definition.

Equations (1)–(2) depend on the background $[\tau_0,t_0]$, which is why it is required that the functions $x_{ij}(\tau) \equiv x_{ij}^0(\tau)$, $\tau \in [\tau_0,t_0]$ and the values $a_j(t_0) = a_j^0 < t_0$, $i = \overline{1,n}$, $j = \overline{1,m}$ are pre-set.

The functions $x_{ij}^0(\tau)$ can be found on the basis of the solution of the existing task concerning the distribution of the existing fleet of combine harvesters by the types of works in each background year $[\tau_0,t_0]$, or at least for the year t_0, which is the start of the scheduled period. The values $a_j(t_0) = a_j^0$ can be found on the basis of the real dynamics of the write-off of the outdated combine harvesters during the previous development history $[\tau_0,t_0]$.

Depending on the correlation of unknown factors and the equations in the models (1) – (2), two kinds of tasks can be resolved with the given model.

If the number of equations coincides with the number of unknown factors, and the task is completely determined, we get the task of a multi-path forecast for the development of the fleet of combine harvesters within a specific scheduled time period $[\tau_0,T]$. If the number of unknowns is higher than the number of equations, we get the task of optimal control within the scheduled interval $[\tau_0,T]$.

In the event of $m_i > 1$ in the models (1)–(2) the number of unknown factors is higher than the number of equations, which is why the solution is not the only one, and for closing the task, it is required to add a certain optimisation criterion. It allows to solve the problem using the models (1) – (2) as the task of the optimal control over the demand and the renewal of the fleet of combine harvesters within the scheduled interval $[t_0,T]$.

Taking into account the increasing deficit of labour resources in agriculture, we will consider the demand for labour force required for the performance of pre-set tasks within the scheduled interval $[t_0,T]$ as one of the main components of the optimisation criteria. In other words, we will be using the function $P(t)$, represented by the formula (2), as a component included in the optimisation criterion, omitting the limitation (2) in the models (1) – (2). We are also going to use the operating expenses for the performance of the specific volume of works and operations and the expenses for the acquisition of harvester-threshers as the components included in the optimisation criterion.

Thus, we are arrive at the definition of the general problem regarding the optimal control over the demand and the renewal of the fleet of combine harvesters – it involves the tasks of minimising labour costs, operating expenses, and the expenses for the acquisition of the new grain harvesting machines and equipment.

To determine the unknown functions $x_{ij}(t)$, $a_j(t)$, $i = \overline{1,n}$, $j = \overline{1,m_i}$, $t \in [t_0, T]$, which provide the minimum of the function:

$$I = \sum_{i=1}^{n} \sum_{j=1}^{m_i} \int_{t_0}^{T} \left\{ \int_{a_j(t)}^{t} \left[P_{ij}(\tau) + r_{ij}(\tau,t) \right] x_{ij}(\tau) d\tau + B_j(t) x_j(t) \right\} dt \to \min, \tag{3}$$

with equality constraints:

$$S_i(t) = \sum_{j=1}^{m_i} \int_{a_{j(t)}}^{t} \beta_{ij}(\tau,t) x_{ij}(\tau) d\tau, \tag{4}$$

$$i = \overline{1,n}$$

with inequality constraints:

$$x_{ij} \geq = 0, \qquad a_j(t) < t, \qquad i = \overline{1,n} \qquad j = \overline{1,m_i} \tag{5}$$

under the initial conditions:

$$a_j(t_0) = a_j^0 \geq 0, \qquad x_{ij}(\tau) \equiv x_{ij}^0(\tau),$$

$$i = \overline{1,n}, \quad j = \overline{1,m_i}, \qquad \tau \in [\tau_0, t_0], \tag{6}$$

where $r_{ij}(\tau,t)$ are operating costs per unit (labour expenses, fuel costs, allocations for in-line repair works and technical maintenance, etc.) for harvesting the i crop in t per one harvester of the j make, purchased in τ; $B_j(t)$ is the cost of the combine harvester of the j make which will be purchased in t, $\boldsymbol{j = \overline{1,m_i}}$, $\tau \in [t_0, T]$.

3. Results and Discussion

In this task, the functions $S_i(t)$, $\beta_{ij}(\tau,t)$, $P_{ij}(\tau)$, $r_{ij}(\tau,t)$, $B_j(t)$ are always known. after determining $x_{ij}(t)$ the required number of harvester-threshers of the j make can be obtained:

$$x_j(t) = \sum_{i=1}^{n} x_{ij}(t),$$
$$j = \overline{1,m}.$$

After determining the functions $a_j(t)$, $j = \overline{1,m}$ all of the combine harvesters of the j make which were purchased in τ can be written off in t, provided that $\tau < a_j(t)$. Besides, the service life of combine harvesters $d_j(t)$ can be determined with the functions $d_j(t) = t - a_j(t)$, $j = \overline{1,m}$.

In the optimal control task (3)–(6) at $m_i > 1$ the number of unknowns $x_{ij}(t)$, $a_j(t)$, $i = \overline{1,n}$, $j = \overline{1,m_i}$, is higher than the number of equations. Thus, it is quite complicated to resolve this task in general. This is why we are making the assumption that $m_i = 1$, i.e., that every i^{th} crop is harvested by harvester-threshers of the same type. Since we have assumed that there is a small number of different makes of harvesters in the same country, such an assumption is quite acceptable from a practical standpoint.

When $m_i = 1$, the task (3) – (6) is in the following form.

To determine the unknown functions $x_i(t)$, $a_i(t)$, $i = \overline{1,n}$, $t \in [t_0, T]$, providing

$$I = \sum_{i=1}^{n} \int_{t_0}^{T} \left\{ \int_{a_j(t)}^{t} [P_i(\tau) + r_i(\tau,t)]\, x_i(\tau) d\tau + B_i(t) x_i(t) \right\} dt \to \min, \tag{7}$$

with equality constraints:

$$S_i(t) = \int_{a_{j(t)}}^{t} \beta_i(\tau,t) x_i(\tau) d\tau, \tag{8}$$
$$i = \overline{1,n},$$

with inequality constraints:

$$0 \le x_i(t) \le M_i, \qquad a_i'(t) \ge 0, \qquad 0 \le a_i(t) \le t, \qquad i = \overline{1,n}, \tag{9}$$

under the initial conditions:

$$a_i(t_0) = a_i^0 \ge 0, \qquad x_i(\tau) \equiv x_i^0(\tau), \quad \tau \in [\tau_0, t_0], \qquad i = \overline{1,n}, \tag{10}$$

while the functions $a_i(t)$ are regarded as different from each other. We will assume that the pre-set functions are the following:

$$\beta_i(\tau,t), \quad r_i(\tau,t) \in C_{[\tau_o,T]\otimes[t_o,T]}, \quad B_i(t) \in C_{[t_o,T]}, \quad S_i(t) \in C^1_{[t_o,T]},$$
$$x_i^o(\tau) \in C_{[\tau_o,t_o]}, \quad P_i(\tau) \in C_{[t_o,T]},$$

all of the functions are positive, and the initial values (10) comply with the equation system (8) at $t = t_0$.

It turns out that this task can be regarded as the group n of optimisation tasks for single-product models of the following type.

To determine the unknown functions, $x_i(t)$, $a_i(t)$, $t \in [t_0, T]$,
where i is fixed, providing

$$I = \int_{t_0}^{T} \left\{ \int_{a_i(t)}^{t} [P_i(\tau) + r_i(\tau,t)]\, x_i(\tau) d\tau + B_i(t) x_i(t) \right\} dt \to \min, \tag{11}$$

with limitations (8)–(10) for the relevant i.

It is obvious that $I = \sum_{i=1}^{n} I_i$, while $I \to \min$, if $I_i \to \min$, $i = \overline{1,n}$.

4. Conclusions

The qualitative analysis of the optimisation task provided in the present paper allows to determine the optimum service life of combine harvesters on the basis of the main properties of the solutions, which require much less information in comparison with the direct solution of an optimisation task. The modes that are obtained in the result of resolving integro-functional equations show the main regularities of the dynamics of renewal and development of the fleet of combine harvesters. The lack or unreliability of the source data, which is quite usual in agriculture, justifies the application of the mainstream approach to studying issues connected with the replacement and renewal of combine harvesters.

References

Bochtis, D., Sørensen, C.G.C., Busato, P., 2014. Advances in agricultural machinery management: A review. Biosystems Engineering, 126, 69-81.

Bochtis, D., Vougioukas, S., Ampatzidis, Y., Tsatsarelis, C., 2007. On-line co-ordination of combines and transport carts during harvesting operations. 6th Eurpean Conf. on Precision Agriculture, ECPA 2007, 715-721.

Edwards,W., Boehlje, M., 1980. Machinery selection considering timeliness losses. Transactions of the American Society of Agricultural Engineers, 23 (4), 810-815.

Gao, Huan-Wen, Hunt, Donnell R., 1985. Optimum combine fleet selection with power-based models. Transactions of the American Society of Agricultural Engineers, 28 (2), 364-368.

Gunnarson, C., Spörndly, R., Rosenquist, H., De Toro, A., Hansson, P.-A., 2009. A method of estimating timeliness costs in forage harvesting illustrated using harvesting systems in Sweden. Grass and Forage Science, 64 (3), 276-291.

Miu, P.I., Beck, F., Kutzbach, H.D., 1997. Mathematical modelling of threshing and separating process in axial threshing. ASAE Paper No. 97-1063, St. Josep, MI.

Trollope, J.R., 1982. A mathematical model of the threshing process in a conventional combine-thresher. Journal of Agricultural Engineering Research, 27 (2), 119-130.